

MAS 117

Introduction to Algebra

Assignment 3

For handing in on 31 January 2007

*Write your name and student number at the top of your assignment before handing it in. Staple all the pages together. Post the assignment in the **RED** post-box on the **BASEMENT** of the Maths building before 16:00 on Wednesday.*

1 (15 marks) Find two 2×2 matrices A and B , having entries 0 and 1 only, such that $AB \neq BA$.

2 (25 marks) Let A be a 2×2 matrix, and suppose that $\det(A - xI_2) = x^2 + ax + b$. Show that

$$A^2 + aA + bI_2 = O_{2 \times 2}.$$

3 (35 marks) The *trace* of a square matrix A , written $\text{Trace}(A)$, is the sum of the elements on the main diagonal of A .

(a) (10 marks) Show that, if A and B are 2×2 matrices, then $\text{Trace}(AB) = \text{Trace}(BA)$.

(b) (10 marks) Do there exist 2×2 matrices A and B such that $AB - BA = I_2$?

(c) (15 marks) Extend your conclusions to matrices of arbitrary size.

4 (25 marks) The *symmetric difference* $A \triangle B$ of two sets A and B is defined to be the set consisting of elements lying in either A or B but not both.

(a) (20 marks) Show (using Venn diagrams if you like) that symmetric difference satisfies the commutative, associative, identity, and inverse laws. What is the identity? What is the inverse of A ?

(b) (5 marks) Show that symmetric difference and intersection satisfy the distributive law:

$$(A \triangle B) \cap C = (A \cap C) \triangle (B \cap C).$$

5 Prize question A cash prize is offered for the best solution received to this question. Do not hand this in with the rest of your coursework; give your solutions to me.

Let X be the set of all 2×2 matrices of the form $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$, where a and b are real numbers.

- (a) Show that the sum or product of two matrices in X is also in X .
- (b) What is the connection between X and the set of complex numbers?